Applications of Logarithms

Logarithms
Algebra II

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Abstract

By now, we've learned how to evaluate logarithms using several rules and properties. Finally, we're going to put all of these things together and learn how to evaluate more complicated equations with logarithms, which also see usage in our everyday lives.

1 Introduction

When solving exponential equations with logarithms, the best advice is to isolate the exponential term as if it were a variable, and solve it by converting to logarithmic form. We're going to cover how to solve two types of exponential equations:

2 Form $a \cdot b^x = d$

Let's say we wanted to solve

$$5 \cdot 2^x = 240$$

To start, we would move all of the numbers to one side

$$2^x = 48$$

Now, we would solve for x by converting to logarithmic form

$$\log_2(48) = x$$

Since we only have a calculator with log and ln buttons, we can change the base of this logarithm to evaluate it

$$x = \frac{\log(48)}{\log(2)}$$
$$x \approx 5.585$$

2.1 Questions

1. Convert $2 \cdot 6^x = 236$ to logarithmic form
2. Solve $5 \cdot 3^t = 20$. *Round your answer to the nearest thousandth*
3. Solve $6 \cdot e^y = 300$. *Round your answer to the nearest thousandth*

3 Form $a \cdot b^{cx} = d$

Let's do another example, this time solving

$$6 \cdot 10^{2x} = 48$$

Just like last time, isolate the exponent

$$10^{2x} = 8$$

Now, we can convert to logarithmic form

$$\log_{10}(8) = 2x$$

To get the exact answer, we can isolate x by dividing everything by 2

$$\frac{\log_{10}(8)}{2}$$

We know that $\log_{10}(x) = \log(x)$ on our calculator, so we can just evaluate the equation

$$x \approx 0.452$$

3.1 Questions

1. Convert $3 \cdot 10^{4t} = 522$ to logarithmic form
2. Solve $4 \cdot 5^{2x} = 300$
3. Solve $-2 \cdot 3^{(0.2z)} = -400$
4. (Challenge) What are the solutions to $(2^x - 3)(2^x - 4) = 0$

4 Extra Practice:

Courtesy of KhanAcademy:

1. Solve exponential equations using logarithms: base-10 and base-e
2. Solve exponential equations using logarithms: base-2 and other bases
3. Exponential word problems

5 Answer Key

Form $a \cdot b^x = d$:

1. $x = \log_6(118)$
2. $t \approx 1.262$
3. $y \approx 3.912$

Form $a \cdot b^{cx} = d$:

1. $t = \frac{\log(174)}{4}$
2. $x \approx 1.341$
3. $z \approx 24.114$
4. $\log_2(3)$ and 2