Intro to Parabolas

Quadratic Functions and Equations
Algebra 1

Julian Zhang

April 2021

Abstract

This handout is a transcription and synthesis of the content covered in the Quadratic Functions and Equations unit of Algebra 1 from KhanAcademy. In this handout, we will be introducing parabolas and covering how to solve and graph them with factored form. Content and questions are courtesy of Sal Khan and KhanAcademy. The images are not mine; they are the property of their respective owners.

1 Intro to Parabolas

A parabola is one of the most common curves in mathematics. Its name comes from the Greek work *para*, meaning something alongside or something in **parallel**. The word *bola* refers to ballistics, or throwing something. We can now interpret the word parabola as being alongside something being thrown.

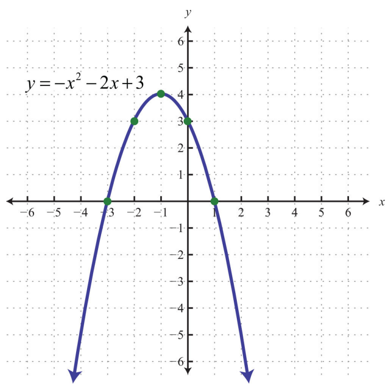

Figure 1: A basic parabola

At first glance, this parabola might look like an arch. While that is a good way to visualize it, I implore you to think of it in a different way. Imagine you were standing on a beach, and threw a perfectly spherical rock into the air. If you were to look at that rock being thrown from far away (in a perfect world with no air resistance, the rock would travel in a parabola, hence it's Greek name. Note that other parabolas exist too, with various orientatons.

Per the graph above, try graphing the equation $y = x^2$ using the values -3, -2, -1, 0, 1, 2, and 3. Now, we are able to step aside from the arch or stone throw analogy, and properly define a parabola as **the graph of a function with degree 2.**

Parabolas also have what we call a vertex, that being the highest point (maximum) on a parabola facing downwards, or the lowest point on a parabola facing upwards (minimum). One

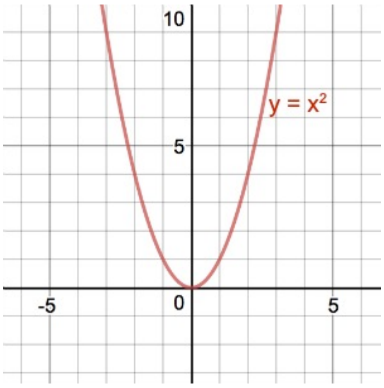

Figure 2: A parabola facing upwards, the graph of $y = x^2$

might also notice that parabolas are always symmetrical. This means that there is always a certain line on which we can fold the parabola on itself. Mathematicians call this the axis of symmetry.

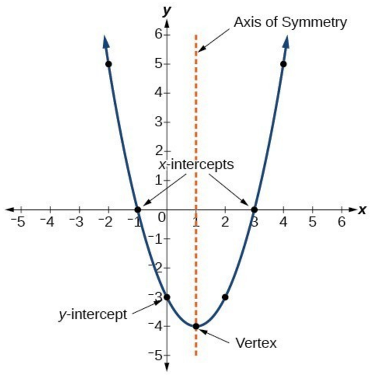

Figure 3: Axis of Symmetry of a parabola

Most parabolas will intercept the axes thrice: the x-axis twice, and the y-axis once. It's possible for a parabola to intercept the x-axis only once or even not at all, but for the purposes of this lesson, we will focus on when there are two x-intercepts. An astute learner would realize that due to the symmetrical nature of a parabola, the axis of symmetry falls in the middle of the two x-intercepts. As such, we can derive a formula for the axis of symmetry. Given that the x-intercepts are at $(x_0, 0)$ and $(x_1, 0)$, the formula for the axis of symmetry is:

$$x = \frac{x_0 + x_1}{2}$$

1.1 Questions

1. Given a parabola with x-intercepts 0 and 4, determine its axis of symmetry.
Solution: Using the previously obtained formula, we plug in the x-intercepts. This gives us:

$$x = \frac{0 + 4}{2}$$

$$x = \frac{4}{2}$$

$$x = 2$$

2. Given a parabola with x-intercepts (-7, 0) and (-3, 0), determine its axis of symmetry.
3. Given a parabola with x-intercepts -9 and 1, determine its axis of symmetry.

4. Given a parabola with x-intercepts -2 and 2, determine its axis of symmetry.
5. A parabola has vertex (1, 4), and intercepts the x-axis at -5. Determine its other x-intercept.
6. A downward facing parabola has one x intercept at (3, 0) and reaches it's highest point at (-1, 16). Determine its other x-intercept.

Extra Practice (Courtesy of KhanAcademy): Parabolas intro, Interpret parabolas in context (application), Interpret a quadratic graph (application)

2 Solving and Graphing with factored form

2.1 Null factor law

Take the quadratic equation $(2x-1)(x+4)=0$. Firstly, let's prove that it's a quadratic by expanding it with the FOIL (**F**irst, **I**nner, **O**uter, **L**asts) method.

$$2x(x)+2x(4)-1(x)-1(4)=0$$
$$2x^2+8x-x-4=0$$
$$2x^2+7x-4=0$$

We don't need to solve the equation this way, we just expanded it to prove that it is indeed a quadratic, as it's highest degree term ($2x^2$) is degree 2. Going back to our original equation, $(2x-1)(x+4)=0$, I want to introduce something known as the Null factor law or Zero product property. This states, **for any two integers a and b who's product is 0, either a, b, or both a and b are equal to zero.**

$$a \times b = 0$$
$$\text{Either a or b or both must} = 0.$$

This should be pretty intuitive, as the properties of zero are that of "nothing", and you can only get "nothing" by multiplying "nothing" by something. Let's try and solve $(2x-1)(x+4)=0$ with the Null factor law, with $a=2x-1$ and $b=x+4$. For ease of calculation, we'll break this down into two cases, when a and b respectively equal zero:

Case 1: $2x-1=0$
We can solve this like a regular linear equation. Start by adding 1 to both sides:

$$2x=1$$

Divide both sides by 2:

$$x=\frac{1}{2}$$

And that is our solution to Case 1.

Case 2: $x+4=0$
Subtract both sides by 4

$$x=-4$$

And that is our solution to Case 2.
Note that theoretically, you should also check for the case where both a and b are equal to 0, but given that a and b are different, this third case can be disregarded.

We can now conclude that the solution to the equation $(2x-1)(x+4)=0$ is $x=\frac{1}{2}$ ***OR*** $x=-4$ (Yes, the equation has two solutions). Try these practice problems, and in the next section we will explain what these solutions mean when graphing an equation.

2.2 Questions

1. Given $f(x) = (x-5)(5x+2)$, for what values of x does $f(x) = 0$?
Solution: $x - 5 = 0$, or $5x + 2 = 0$
Case 1: $x - 5 = 0$

$$x = 5$$

Case 2: $5x + 2 = 0$

$$5x = -2$$
$$x = \frac{-2}{5}$$

Answer: $x =$ or $x = \frac{-2}{5}$

2. Solve for x: $(4x+1)(-2x+3) = 0$
3. Solve for x: $(x+5)(2x-1) = 0$
4. Solve for x: $(-x-1)(8x-9) = 0$
5. Given $f(x) = (6x+2)(x+4)$, for what values of x does $f(x) = 0$?
6. Given $f(x) = (x+1)^2$, for what values of x does $f(x) = 0$?

Extra Practice (Courtesy of KhanAcademy): Zero product property.

2.3 Graphing quadratics in factored form

In order to learn how to graph a parabola, we'll take a look at another equation, this time expressed in terms of x and y so we can graph it on a Cartesian plane:

$$y = \frac{1}{2}(x-6)(x+2)$$

Again, we know that this equation is a parabola because when expanded, it's highest term will contain an x^2. The easiest way to graph any parabola is to find the x intercepts using the method outlined in the previous chapter. Since x intercepts occur when the graph touches the x axis (hence when the y value is 0), we'll set y to be 0, then solve.

$$0 = \frac{1}{2}(x-6)(x+2)$$

In this particular case, we can actually divide both sides by $\frac{1}{2}$, which is the equivalent of multiplying both sides by 2. Since $\frac{0}{2} = 0$, this doesn't change much, and we are left with

$$0 = (x-6)(x+2)$$

Applying the Null factor law, we get that either $x - 6 = 0$, or $x + 2 = 0$. Thus, the solutions, or x-intercepts of this parabola are 6 and -2. This means that the parabola crosses the x-intercepts at $(6, 0)$ and $(-2, 0)$. To go even further, we can calculate the axis of symmetry of the parabola using the formula we obtained in the first chapter:

$$x = \frac{6 + (-2)}{2}$$
$$x = 2$$

This means that the parabola is symmetrical when folded over on the line $x = 2$, and that the vertex (in this case, lowest point on the parabola) occurs at $x = 2$. We can now solve for the vertex by plugging in $x = 2$ to the original quadratic.

$$y = \frac{1}{2}(2-6)(2+2)$$

$$y = \frac{1}{2}(-4)(4)$$
$$y = \frac{1}{2}(-16)$$
$$y = -8$$

Finally, we've determined three points on the parabola: two x-intercepts at $(6, 0)$ and $(-2, 0)$, as well as a vertex at $(2, -8)$. We are now able to graph the parabola with these three points, which gives us:

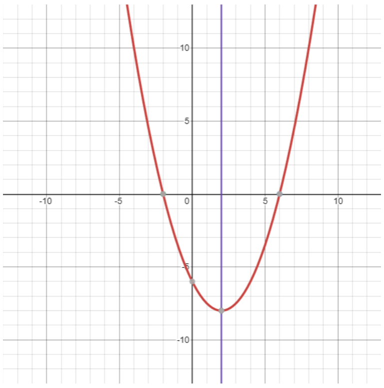

Figure 4: Graph of $y = \frac{1}{2}(x - 6)(x + 2)$

2.4 Questions

(Courtesy of KhanAcademy): Graph quadratics in factored form.
(Courtesy of KhanAcademy): Quadratic word problems (factored form)(applicaton).